5. The Lebesgue outer measure of an interval is its length (see ppg. 31-33). $[0,1]$ is a closed, bounded interval, so its outer measure is $1-0=0$. The Lebesgue outer measure of a countable set is 0, by the following line of reasoning:
Let $\{a_k\}_{k=1}^{\infty}$ be a countable set. Let $\epsilon > 0$ and define $\{I_k\}_{k=1}^{\infty}$ such that $I_k = (a_k - 2/\epsilon^{k+1}, a_k + 2/\epsilon^{k+1})$. Hence, $\sum_{k=1}^{\infty} \ell(I_k) = \epsilon$ and, since ϵ is arbitrary,

$$m^*(\{a_k\}_{k=1}^{\infty}) = \inf\{\sum_{k=1}^{\infty} \ell(I_k) | \cup_{k=1}^{\infty} I_k \supseteq [0,1]\} = 0.$$

Therefore, it is impossible that $[0,1]$ is countable.